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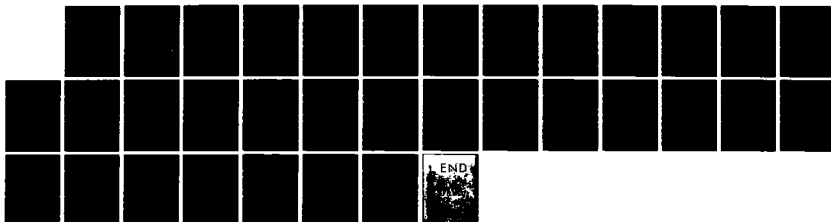
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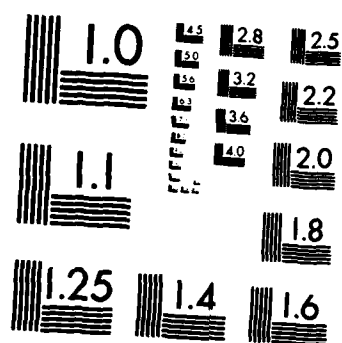
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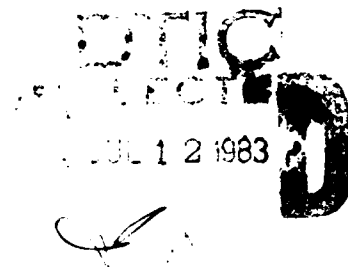
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TRANSVERSE SHEAR EFFECTS IN BIMODULAR
COMPOSITE LAMINATES

by

C.W. Bert and F. Gordaninejad

April 1983



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COMPOSITE LAMINATES

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ABSTRACT

→ A closed-form solution for the Timoshenko-type shear correction coefficient (K_c^2) governing the deflection of bimodular composite laminates in cylindrical bending is presented. The bending-stress distribution for a laminate constructed of bimodular materials (which have different elastic moduli in tension and compression) is used in the two-dimensional equilibrium equation to obtain the transverse shear-stress distribution. This shear-stress distribution is used to obtain expressions for the shear correction coefficient (based on equivalent shear strain energy) and the maximum dimensionless transverse shear stress ($\bar{\tau}_{xz}$)_{max}. Finally, the effects of the elastic-constant ratios on the neutral-surface location and shear correction coefficient for laminates consisting of either unimodular or bimodular materials are studied. ←

$(\bar{\tau}_{xz})_{max}$

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INTRODUCTION

Materials which have different moduli in tension and in compression are called bimodular materials. Rock, concrete, cord-rubber, paperboard, and certain biological tissues are examples of such materials. Even aramid-fiber, polymer-matrix composites exhibit some bimodularity. The analysis of laminated bimodular material is more complicated than unimodular material (ordinary material) due to the dependency of the material stiffness on the material properties, which indeed depend on the state of stress (i.e., tensile or compressive) in the laminate.

Although transverse shear deformations have been considered in the analyses of bimodular laminates [1-4] in recent years, there has been no effort to include the effect of bimodularity of the material on the Timoshenko-type shear correction coefficient, k^2 . In other words, it has been tacitly assumed that the value of k^2 is the same as that of an ordinary-material laminate. The use of such a shear correction factor in predicting the deflection of shear deformable, ordinary-material laminates in cylindrical bending is well established [5,6]. It is noted that Whitney [5] extended Chow's symmetric-laminate work [7] to arbitrary laminates. Also, it can be shown that the resulting expressions in [5,6] are algebraically equivalent.

In this paper, a straight-forward approach analogous to that used in elementary shear theory for single-layer ordinary materials is employed. It may be considered to be a generalization of the work of Bert [6] from ordinary-material laminates to bimodular-material laminates.

THEORY AND FORMULATION

Consider a rectangular-cross-section laminated beam of thickness h . The origin of the Cartesian coordinate system is located on the mid-surface of

the beam with the x axis being along the length of the beam and the z axis being measured positive downward. The same displacement field as used in shear deformable beam theory is implemented here:

$$\begin{aligned} u(x,z) &= u^0(x) + z\psi(x) \\ w(x,z) &= w^0(x) \end{aligned} \quad (1)$$

The axial normal strain and transverse shear strain are given by

$$\epsilon_x = u^0_{,x} + z\psi_{,x} ; \quad \gamma_{xz} = \psi + w^0_{,x} \quad (2)$$

where a comma denotes the derivative with respect to the quantity following it. The longitudinal bending stress at any distance z from the mid-plane of the laminate is

$$\sigma_x = E^{(k)} \epsilon_x \quad k=c,t \quad (3)$$

Also

$$\tau_{xz} = G^{(k)} \gamma_{xz} \quad (4)$$

The longitudinal stress resultant and stress couple are defined as

$$(N,M) = \int_{-h/2}^{h/2} (1,z) \sigma_x dz \quad (5)$$

With the aid of Equations (2), (3), and (5), one can derive the following laminate constitutive relation

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} u^0_{,x} \\ \psi_{,x} \end{Bmatrix} \quad (6)$$

where

$$(A,B,D) = \int_{-h/2}^{h/2} E^{(k)} (1,z,z^2) dz \quad k=c,t$$

In the absence of body force, the two-dimensional static equilibrium equation for forces in the direction along the length of the beam is

$$\sigma_{x,x} + \tau_{xz,z} = 0 \quad (7)$$

Integrating Equation (7) with respect to z and using Equations (3) and (4), one has

$$\gamma_{xz} = - [G^{(k)} (AD - B^2)]^{-1} [(Da - Bb)N_{,x} + (Ab - Ba)M_{,x}] \quad (8)$$

where a and b are "partial stiffnesses" for stretching and bending-stretching coupling defined by the following expressions for the convex bending case (top layers in compression and bottom layers in tension)

$$(a,b) = \begin{cases} \int_{-h/2}^z (1,z) E^c dz & z_n > z > -h/2 \\ \int_{-h/2}^{z_n} (1,z) E^c dz + \int_{z_n}^z (1,z) E^t dz & h/2 > z > z_n \end{cases} \quad (9)$$

Multiply Equation (7) by unity and z , respectively, and then integrating through the thickness, one obtains the following equilibrium equations, which coincide with those of elementary beam theory,

$$M_{,x} = Q, \quad N_{,x} = 0 \quad (10)$$

where

$$Q = \bar{\gamma}_{xz} \int_{-h/2}^{h/2} G^{(k)} dz \quad (11)$$

Therefore, Equation (8) simplifies as follows:

$$\gamma_{xz} = -Q(Ab - Ba)[G^{(k)}(AD - B^2)]^{-1} \quad (12)$$

Extending the definition of the shear correction coefficient as used in [6], which is based on the shear strain energy, to the case of bimodular material, one has

$$K^2 = \frac{(AD - B^2)^2}{\left[\int_{-h/2}^{h/2} G^{(k)} dz \right] \left[\int_{-h/2}^{h/2} (1/G^{(k)})(Ab - Ba)^2 dz \right]} \quad (13)$$

If G is constant throughout the thickness of the beam, Equation (13) can be simplified to

$$K^2 = \frac{(AD - B^2)^2}{h \int_{-h/2}^{h/2} (Ab - Ba)^2 dz} \quad (14)$$

By changing the coordinate from z measured from the midplane to z' measured from the neutral-surface position, one can simplify Equation (14) as follows:

$$K^2 = \frac{D^2}{h \int_{-h_c}^{h_t} b^2 dz'} \quad (15)$$

To evaluate the stiffnesses (A, B , and D), one needs to know about the location of the neutral-surface position. A closed-form solution can be obtained only when the neutral-surface position (z_n) and thus, the stiffnesses do not depend on x . The same criterion to define z_n as used in [4] is applied here

$$\epsilon_x = 0 \quad (16)$$

Using Equations (2) and (5) and assuming* that

$$N = 0 \quad (17)$$

one gets

$$z_n = B/A = \text{constant} \quad (18)$$

Equation (18) can be solved explicitly for z_n .

It is convenient to introduce the dimensionless transverse shear stress defined as follows:

$$\bar{\tau}_{xz} = \frac{\tau_{xz}}{Q/h} \quad (19)$$

Combining Equations (4) and (12) and measuring z' from the neutral-surface location, one obtains

$$\bar{\tau}_{xz} = bh/D \quad (20)$$

Detailed derivations for K^2 and $(\bar{\tau}_{xz})_{\max}$ are given in the Appendix.

NUMERICAL RESULTS

Three cases of laminates constructed of bimodular materials are considered here: single-layer, two-layer, and three-layer. Numerical results obtained from closed-form solutions for the shear correction coefficient and the maximum dimensionless shear stress for different bimodular-material parameters are presented. Also, the effects of elastic-constant ratios on neutral-surface position and K^2 for both unimodular and bimodular laminates (up to three layers) are studied. The laminates are assumed to be made of equal-thickness layers and the effect of the sign of the longitudinal normal

*Note that this assumption is made only for the bimodular case (not unimodular).

strain on shear moduli is ignored (that is, $G^c/G^t = 1$).

In the case of single-layer bimodular material (see the Appendix), it is interesting to note that both the transverse shear correction coefficient and the maximum transverse shear stress are unaffected by the bimodular ratio E^c/E^t . However, this does not imply that the distribution of shear stress is the same for unimodular and bimodular materials (see Fig. A-1). The effect of bimodular ratio on the location of the neutral surface is studied in Table 1. For values of $E^c < E^t$ (e.g., aramid-cord/rubber), h_t falls within the lower half of the thickness; whereas for materials such as rock ($E^c > E^t$), h_t is in the upper half of the beam (for the convex bending case).

In bending of a two-layer bimodular laminate since one of the two layers is always in compression (or tension), only three of the four elastic moduli belonging to these layers are pertinent [see Fig. A-2]. This means that the location of the neutral surface depends on only two elastic-moduli ratios. The effects of these two ratios on h_t are studied for a wide range of E_2^c/E_2^t in Table 2. The shear correction coefficient varies drastically with the ratio of the shear moduli of the two layers (G_1/G_2), whereas it changes only a little for a wide range of E_2^c/E_2^t for $E_1^c/E_2^t = 1$, as shown in Fig. 1.

The shear correction coefficient for two-layer unimodular laminates increases rapidly from 50% to 98% of the classical value (5/6) as the elastic-modulus ratio E_1/E_2 varies from 0 to 0.25 as shown in Fig. 2. For ratios greater than 0.5, a value of 5/6 would be a good approximation for K^2 . The dimensionless maximum shear stress increases from 0 to 12 as E_1/E_2 changes from 0 to infinity as tabulated in Table 3.

Three-layer bimodular laminates which have facings (top and bottom layers) made of the same material are considered here. For the case in which $E_1^c/E_2^c = 1$, the effect of bimodular ratio E_3^t/E_2^c on h_t/h for different ratios of E_2^t/E_2^c (1/2, 1, and 2) are investigated in Table 4. As E_3^t/E_2^t increases from 0 to 10, h_t/h decreases for all three values of E_2^t/E_2^c . Also, a plot of K^2 vs E_1^c/E_2^c is given in Fig. 3 which shows the shear correction factor as a function of the bimodularity of the material.

Finally, the effect of elastic moduli ratio E_2/E_1 on K^2 and $(\bar{\tau}_{xz})_{\max}$ are studied (see Fig. 4 and Table 5) for three-layer unimodular-material laminates with identical facings. For different values of G_1/G_2 , K^2 increases rapidly for E_1/E_2 less than 0.05 but after this point, K^2 changes very slowly. The maximum dimensionless shear stress varies from 0 to 40.5 for $0 < E_1/E_2 < \infty$.

CONCLUSIONS

Closed-form solutions for the Timoshenko-type shear correction coefficient and the maximum dimensionless transverse shear stress are presented for bimodular laminates undergoing cylindrical bending. These solutions depend on bimodular ratio(s) for different laminations. However, it is interesting to note that both the transverse shear correction coefficient and the maximum dimensionless transverse shear stress $(\bar{\tau}_{xz})_{\max}$ for a single-layer material are unaffected by bimodular ratio, E^c/E^t .

For two- and three-layer laminates, both K^2 and $(\bar{\tau}_{xz})_{\max}$ depend upon the bimodular ratios and transverse shear moduli ratios. According to the results presented, in some cases K^2 is less than the classical value of 5/6.

ACKNOWLEDGMENTS

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APPENDIX: DETAILED DERIVATIONS

In the following, detailed derivations for unimodular and bimodular laminates are given. Three types of lamination are considered: single-layer, two-layer, and three-layer. In the latter two cases, all layers are assumed to be of the same thickness. Also, in the bimodular cases, the effect of bimodularity on elastic-shear moduli is ignored (since for most of materials $G^t/G^c = 1$). Further, in the bimodular analyses, axial force is assumed to be zero.

1. Single-Layer* (Bimodular Material)

Normal and shear stress distributions for single-layer, bimodular material are shown in Fig. A-1. Since the axial force is taken to be zero,

$$N = \int_{-h_c}^{h_t} \sigma_x dz' = \kappa \left[\int_{-h_c}^0 E^c z' dz' + \int_0^{h_t} E^t z' dz' \right] = 0 \quad (A-1)$$

Using the fact that

$$h = h_t + h_c \quad (A-2)$$

one can integrate Equation (A-1) to obtain

$$h_c/h = \frac{1}{1 + \sqrt{\beta}}, \quad h_t/h = \frac{\sqrt{\beta}}{1 + \sqrt{\beta}} \quad (A-3)$$

where β is the bimodular elastic-moduli ratio defined as

$$\beta = E^c/E^t$$

From Equations (6) and (9), one has

*This case also might be considered to be a two-layer laminate of ordinary materials but with a certain special thickness ratio ($h_1/h_2 = \sqrt{E_2/E_1}$).

$$D = (E^c h_c^3 + E^t h_t^3)/3 \quad (A-4)$$

$$b = \frac{1}{2} \begin{cases} E^c (z')^2 - E^c h_c^2 & -h_c < z' < 0 \\ E^t (z')^2 - E^c h_c^2 & 0 < z' < h_t \end{cases} \quad (A-5)$$

substitution of Equations (A-4) and (A-5) into Equation (15) and using Equation (A-3) leads to

$$K^2 = 5/6 \quad (A-6)$$

Also, using Equations (19), (20), (A-3), (A-4), and (A-5), one has

$$(\bar{\tau}_{xz})_{\max} = 3/2 \quad (A-7)$$

Although the maximum shear stress remains unaffected by bimodularity ratio, the shear-stress distribution does depend on β . Combining Equations (19), (A-3), (A-4), and (A-5) gives the following shear-stress distribution

$$\bar{\tau}_{xz} = \frac{3}{2} \begin{cases} \sqrt{\beta} - \sqrt{\beta} (\sqrt{\beta} + 1)^2 (z/h)^2 & -\frac{1}{\sqrt{\beta} + 1} < z/h < 0 \\ \sqrt{\beta} - \frac{1}{\sqrt{\beta}} (\sqrt{\beta} + 1)^2 (z/h)^2 & 0 < z/h < \frac{\sqrt{\beta}}{\sqrt{\beta} + 1} \end{cases} \quad (A-8)$$

2. Two-Layer Laminates

(a) Bimodular Material

Each individual layer has different elastic moduli in tension and compression. However, since one layer is only in either a tension or a compression state, only three of the four elastic moduli for two-layer analysis come into the picture (see Fig. A-2). A similar procedure to that used for the single-layer case is used here. To determine the bending and the partial bending-stretching coupling stiffnesses, one is able to

obtain the neutral-surface position from the following quadratic equation

$$N = \left[\int_{-h_c}^{-h_c+h/2} E_1^c z' dz' + \int_{-h_c+h/2}^0 E_2^c z' dz' + \int_0^{h_t} E_2^t z' dz' \right] = 0 \quad (A-9)$$

Knowing the neutral-surface location, one can easily determine the following stiffnesses

$$D = (E_1^c [(h/2 - h_c)^3 + h_c^3] + E_2^c (h/2 - h_c)^3 + E_2^t h_t^3) / 3 \quad (A-10)$$

and

$$b = \frac{1}{2} \begin{cases} E_1^c (z')^2 - E_1^c h_c^2 & -h_c < z' < -h_c + h/2 \\ E_2^c (z')^2 - E_2^t h_t^2 & -h_c + h/2 < z' < 0 \\ E_2^t (z')^2 - E_2^t h_t^2 & 0 < z' < h_t \end{cases} \quad (A-11)$$

Substituting Equations (A-10) and (A-11) into Equation (15), one has

$$\kappa^2 = \frac{5}{6} \cdot \frac{16 g_1 [(\alpha + \beta)(h/2 - h_c)^3 + \alpha h_c^3 + h_t^3]^2}{h(g_1 + 1)[3(g_1 \alpha^2 - \beta^2)(h/2 - h_c)^5 - 10(g_1 \alpha^2 h_c^2 - \beta h_t^2)(h/2 - h_c)^3 + 15(g_1 \alpha^2 h_c^4 - h_t^4)(h/2 - h_c) + 8(g_1 \alpha^2 h_c^5 + h_t^5)]} \quad (A-12)$$

where

$$\alpha = E_1^c / E_2^t, \quad \beta = E_2^c / E_2^t \quad \text{and} \quad g_1 = G_1 / G_2$$

Since h_c/h and h_t/h depend upon α and β , κ^2 is a function of g_1 , α , and β for the case of equal-thickness layers. The maximum dimensionless shear stress also is a function of the bimodular ratios (α, β)

$$(\tau_{xz})_{\max} = \frac{3}{2} \cdot \frac{h}{\alpha[(h/2-h_c)^3 + h_c^3] + 3(h/2-h_c)^3 + h_t^3} \begin{cases} \alpha[h_c^2 - (h/2-h_c)^2] & -h_c < z' < -h_c + h/2 \\ h_t^2 & 0 < z' < h_t \end{cases} \quad (A-13)$$

(b) Unimodular Material

A similar lamination as used in part (a) is considered here. The normal and shear stress distributions are shown in Fig. A-3. Since the flexural-extensional coupling stiffness (b) does not vanish, one must use Equation (13) in order to compute K^2 . Let the transverse shear moduli be the same for both layers ($G_2/G_1 = 1$)* and $\alpha = E_2/E_1$. Then, the stiffnesses can be expressed as

$$\begin{aligned} A &= E_1 h(\alpha+1)/2 \\ B &= E_1 h^2(\alpha+1)/8 \\ D &= E_1 h^3(\alpha+1)/24 \end{aligned} \quad (A-14)$$

and

$$(a,b) = E_1 \begin{cases} [(z+h/2), (z^2-h^2/4)/2] & -h/2 < z < 0 \\ [(\alpha z+h/2), (\alpha z-h^2/4)/2] & 0 < z < h/2 \end{cases} \quad (A-15)$$

Substitution of Equations (A-14) and (A-15) into Equation (13) gives

$$K^2 = \frac{5}{6} \cdot \frac{(\alpha^2 + 14\alpha + 1)^2}{2[\alpha^4 + 12\alpha^3 + 102\alpha^2 + 12\alpha + 1]} \quad (A-16)$$

It is interesting to note that K^2 as given by Equation (A-16) gives the same result for $\alpha = c$ and $\alpha = 1/c$, where c is an arbitrary constant. Combining

*This would be true if the individual layers were made of the same orthotropic material but oriented at 0° and 90° , respectively, i.e., a cross-ply laminated.

Equations (4), (12), (19), (A-14), and (A-15) one gets

$$(\bar{\tau}_{xz})_{\max} = \frac{3}{2} \cdot \frac{8\alpha(\alpha+1)}{\alpha^2 + 14\alpha + 1} \quad (\text{A-17})$$

which shows that $(\bar{\tau}_{xz})_{\max}$ depends on elastic-moduli ratio (E_2/E_1) , only.

3. Three-Layer Laminate

(a) Bimodular Material

Consider a three-layer laminate with the top and bottom layers (facings) made of the same material (i.e., $E_1^c = E_3^c$, $E_1^t = E_3^t$) as shown in Fig. A-4 in the convex downward bending position. One assumes that the neutral-surface position is within the middle layer (which happens for most practical cases). The axial force vanishes if

$$\int_{-h_c}^{-h_c+h/3} E_1^c z' dz' + \int_{-h_c+h/3}^0 E_2^c z' dz' + \int_0^{h_t-h/3} E_2^t z' dz' + \int_{h_t-h/3}^{h_t} E_1^t z' dz' = 0 \quad (\text{A-18})$$

Equation (A-18) can be used to determine the neutral-surface position. Having the values of h_t and h_c enables one to calculate k^2 with the same procedure as used for the two-layer case

$$D = \frac{E_2^c}{3} \{ \alpha [h_c^3 + (h/3 - h_c)^3] - (h/3 - h_c)^3 + \beta (h_t - h/3)^3 + \gamma [h_t^3 - (h_t - h/3)^3] \} \quad (\text{A-19})$$

$$\int_{-h_c}^{h_t} G^{(k)} dz = G_2 h [2g_1 + 1] / 3 \quad (\text{A-20})$$

$$\begin{aligned}
\int_{-h_c}^{h_t} (G^{-1})^{(k)} b^2 dz = \frac{(E_2^c)^2}{4G_2} \{ & g_1^{-1} \alpha^2 \left\{ \frac{1}{5} [(h/3 - h_c)^5 + h_c^5] - \frac{2}{3} h_c^2 [(h/3 - h_c)^3 + h_c^3] + h h_c^4 / 3 \right\} \\
& - \left[\frac{1}{5} (h/3 - h_c)^5 + \frac{2}{3} H_c^2 (h/3 - h_c)^3 + (H_c^2)^2 (h/3 - h_c) \right] \\
& + \left[\frac{\beta^2}{5} (h_t - h/3)^5 + \frac{2\beta}{3} H_c^2 (h_t - h/3)^3 + (H_c^2)^2 (h_t - h/3) \right] \\
& + g_1^{-1} \gamma^2 \left\{ \frac{1}{5} [h_t^5 - (h_t - h/3)^5] - \frac{2}{3} h_t^2 [h_t^3 - (h_t - h/3)^3] + h h_t^4 / 3 \right\} \}
\end{aligned}
\tag{A-21}$$

where

$$H_c^2 = (h/3 - h_c)^2 (\alpha - 1) - h_c^2 \tag{A-22}$$

$$\alpha = E_1^c / E_2^c, \quad \beta = E_2^t / E_2^c, \quad \gamma = E_3^t / E_2^c, \quad \text{and} \quad g_1 = G_1 / G_2$$

By substituting Equations (A-19), (A-20), and (A-21) into Equation (15), one can determine K^2 . Also, using Equations (A-19) and the following expression for b_{\max} (the maximum value of partial coupling stiffness b) in the respective regions, one has the maximum dimensionless shear stress by Equation (19).

$$b_{\max} = \frac{1}{2} \begin{cases} L_1 E_1^c & -h_c < z' < -h_c + h/3 \\ L_1 E_1^c - L_2 E_2^c & -h_c + h/3 < z' < 0 \\ L_1 E_1^c - L_2 E_2^c + L_3 E_2^t & 0 < z' < h_t - h/3 \\ L_4 E_1^t & h_t - h/3 < z' < h_t \end{cases}
\tag{A-23}$$

where

$$\begin{aligned}
L_1 &= (h - h_c)^2 - h_c^2 & L_3 &= (h - h_t)^2 \\
L_2 &= L_1 + h_c^2 & L_4 &= L_3 - h_t^2
\end{aligned}
\tag{A-24}$$

(b) Unimodular Material

Shear, bending, and partial stretching-bending stiffnesses for the three-layer symmetric laminate* with equal-thickness layers as shown in Fig. A-5 are ($g_1 = G_1/G_2$, $\alpha = E_1/E_2$)

$$S = \frac{h}{3G_1} (1 + 2g_1) , \quad (A-25)$$

$$D = \frac{E_2 h^3}{324} (26\alpha + 1) , \quad (A-26)$$

and

$$\int_{-h/2}^{h/2} (G^{-1})^{(k)} b^2 dz' = \frac{E_2^2 h^5}{29160} [102\alpha^2 + g_1(120\alpha^2 + 20\alpha + 1)] \quad (A-27)$$

Substituting these stiffnesses into Equation (13) leads to

$$K^2 = \frac{5}{6} \cdot \frac{[26\alpha + 1]^2 g_1}{(1 + 2g_1)[102\alpha^2 + g_1(120\alpha^2 + 20\alpha + 1)]} \quad (A-28)$$

Absolute dimensionless maximum shear stress occurs at the midplane.

Using the same procedure as used before, one obtains

$$(\bar{\tau}_{xz})_{\max} = (\bar{\tau}_{xz})_{z=0} = \frac{3}{2} \cdot \frac{3(8\alpha + 1)}{26\alpha + 1} \quad (A-29)$$

and the shear stress at the interfaces is

$$(\bar{\tau}_{xz})_{z = \pm h/6} = \frac{3}{2} \cdot \frac{24\alpha}{26\alpha + 1} \quad (A-30)$$

It is interesting to note that

$$\frac{(\bar{\tau}_{xz})_{\max}}{(\bar{\tau}_{xz})_{z = \pm h/6}} = 1 + (1/8\alpha) \quad (A-31)$$

*The top and bottom layers have the same properties.

SYMBOLS

a	partial extensional stiffness
A	extensional stiffness
b	partial flexural-extensional coupling stiffness
B	flexural-extensional coupling stiffness
c	arbitrary constant
D	flexural stiffness
E	longitudinal Young's modulus
g	transverse shear moduli ratio
G	transverse shear moduli
h	beam thickness
H_c	defined by Equation (A-22)
K^2	shear correction coefficient
L_i $i=1,2,3,4$	defined by Equation (A-24)
M	moment resultant
N	force resultant
Q	transverse shear resultant
S	thickness-shear stiffness
u,w	displacements in x and z directions
x	coordinate along the length
z	thickness-direction coordinate, measured positive downward from the midplane
z'	thickness-direction coordinate, measured positive downward from neutral-surface position
z_n	neutral-surface location for zero normal strain in the x direction

α, β, γ	bimodular longitudinal Young's moduli ratios
ϵ_x	normal strain in the x direction
$\gamma_{xz}, \tilde{\gamma}_{xz}$	engineering shear strain in xz plane: actual and uniform-shear-distribution cases
τ_{xz}	shear stress in xz plane
σ_x	normal stress in the x direction
ψ	bending slope

Subscript

1,2,3	layer number
$(\)_{,x}$	$\partial(\)/\partial x$

Superscript

o	midplane
-	dimensionless
(c,t)	(compression, tension)
(k)	index (= c,t)

Table 1. Effect of Bimodular Ratio on Neutral-Surface Location for Single-Layer Bimodular Material

E^c/E^t	h_t/h
0	0
1/2	0.414
1	0.500
2	0.586
5	0.691
10	0.760
100	0.909
1000	0.969
∞	1

Table 2. Effect of Bimodular Ratios on Neutral-Surface Location for Two-Layer Bimodular Laminates ($h_1/h_2 = 1$)

E_2^c/E_2^t	h_t/h		
	$E_1^c/E_2^t = 1/2$	$E_1^c/E_2^t = 1$	$E_1^c/E_2^t = 2$
0	0.411	0	0.5811
1/4	0.413	0.333	0.5816
1/2	0.414	0.414	0.5822
3/4	0.415	0.464	0.5827
1	0.417	0.500	0.5833
2	0.421	0.586	0.5858
3	0.424	0.634	0.5886
4	0.427	0.666	0.5917
5	0.430	0.691	0.5955
10	0.439	0.760	0.6667

Table 3. Effect of Elastic-Moduli Ratio on Dimensionless Maximum Shear Stress for a Two-Layer Unimodular-Material Laminate ($h_1/h_2 = 1$)

E_1/E_2	$(2/3)(\bar{\tau}_{xz})_{\max}$
0	0
1/4	0.548
1/2	0.727
3/4	0.870
1	1.000
2	1.456
5	2.791
10	6.241
100	7.087
1000	7.897
∞	8.000

Table 4. Effect of Bimodular Ratios on Neutral-Surface Location for Three-Layer Bimodular Laminates ($h_1/h_2 = h_3/h_2 = 1$, $E_1^c/E_2^c = E_3^c/E_2^c = 1$)

E_3^t/E_2^c	h_t/h		
	$E_2^t/E_2^c = 1/2$	$E_2^t/E_2^c = 1$	$E_2^t/E_2^c = 2$
0	0.724	0.833	0.609
1/4	0.643	0.722	0.573
1/2	0.586	0.633	0.541
3/4	0.542	0.561	0.513
1	0.508	0.500	0.488
2	0.418	0.333	0.414
5	0.310	0.119	0.309
8	0.267	0.033	0.266
10	0.250	0	0.249

Table 5. Effect of Elastic-Moduli Ratio on Dimensionless Maximum Shear Stress for Three-Layer Laminate of Unimodular Materials ($h_1/h_2 = h_3/h_2 = 1$, $E_1/E_3 = 1$)

E_1/E_2	$(2/3)(\bar{\tau}_{xz})_{\max}$
0	0
1/4	0.257
1/2	0.509
3/4	0.757
1	1.000
2	1.928
5	4.355
10	7.500
100	21.428
1000	26.316
∞	27.000

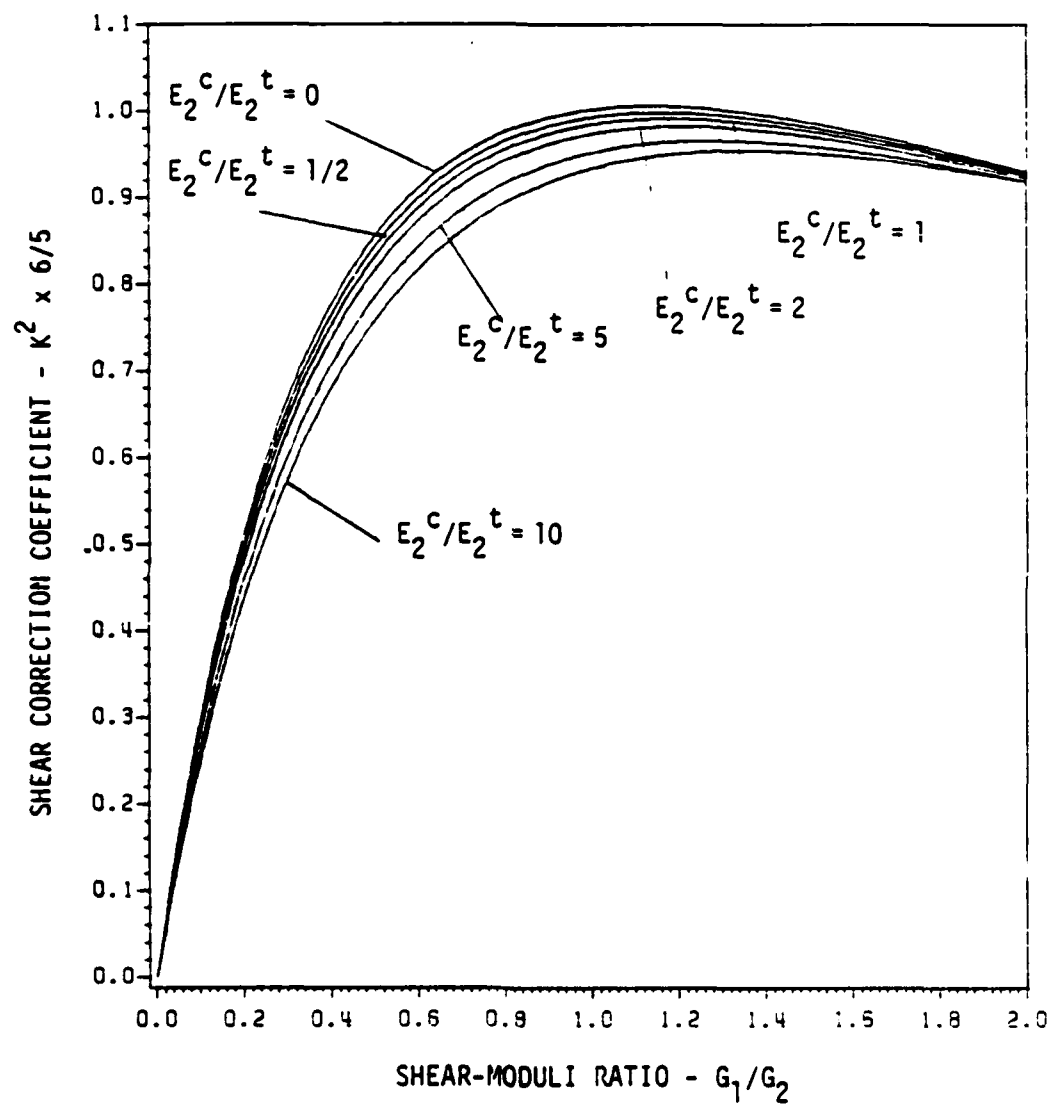


Figure 1. Effect of shear modulus ratio on shear correction coefficient for various bimodulus ratios (E_2^c/E_2^t) for two-layer bimodular laminates ($h_1/h_2 = 1$, $E_1^c/E_2^t = 1/2$).

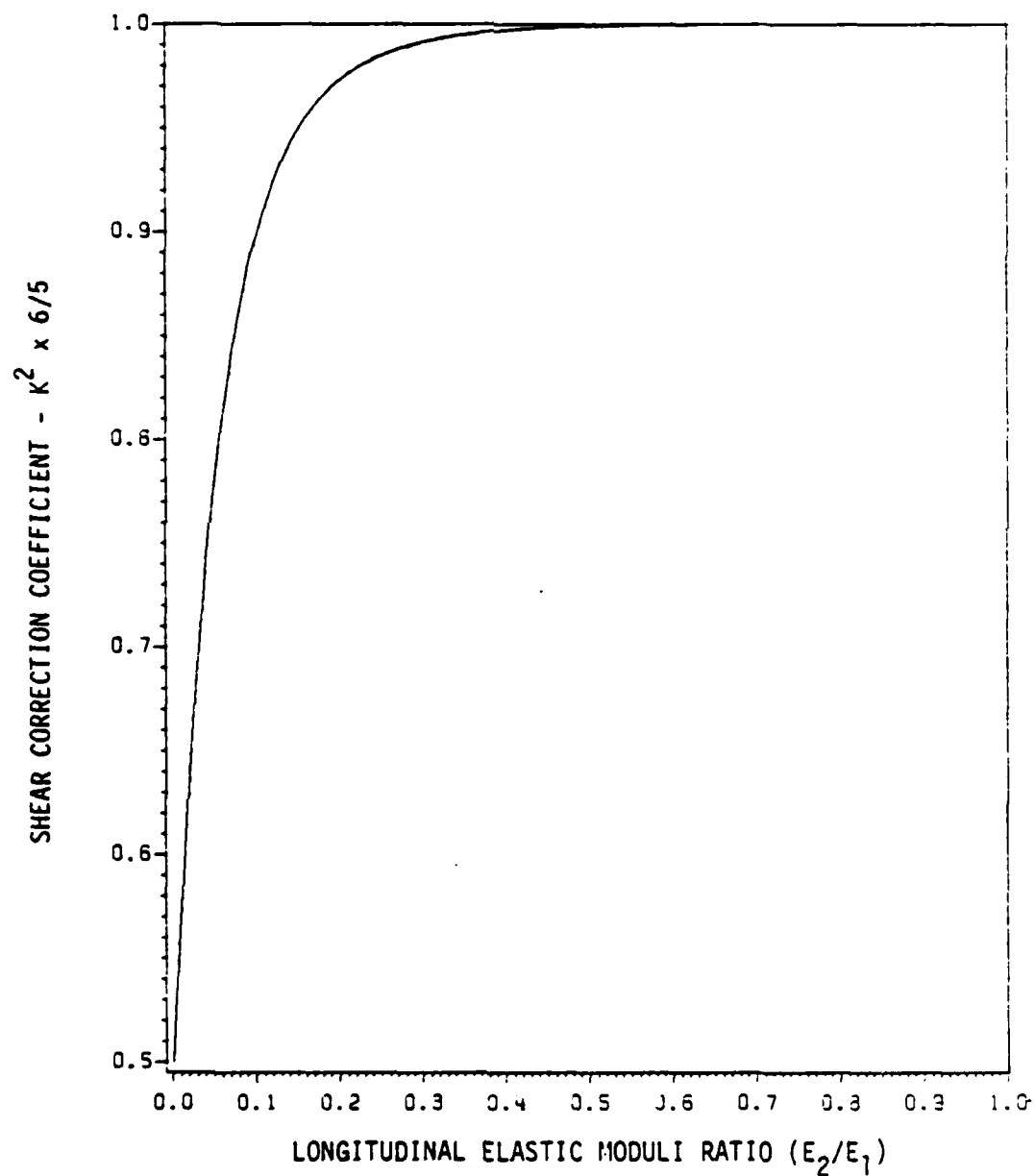


Figure 2. Effect of longitudinal Young's modulus ratio on shear correction coefficient for two-layer unimodular materials ($h_1/h_2 = 1$, $G_1/G_2 = 1$).

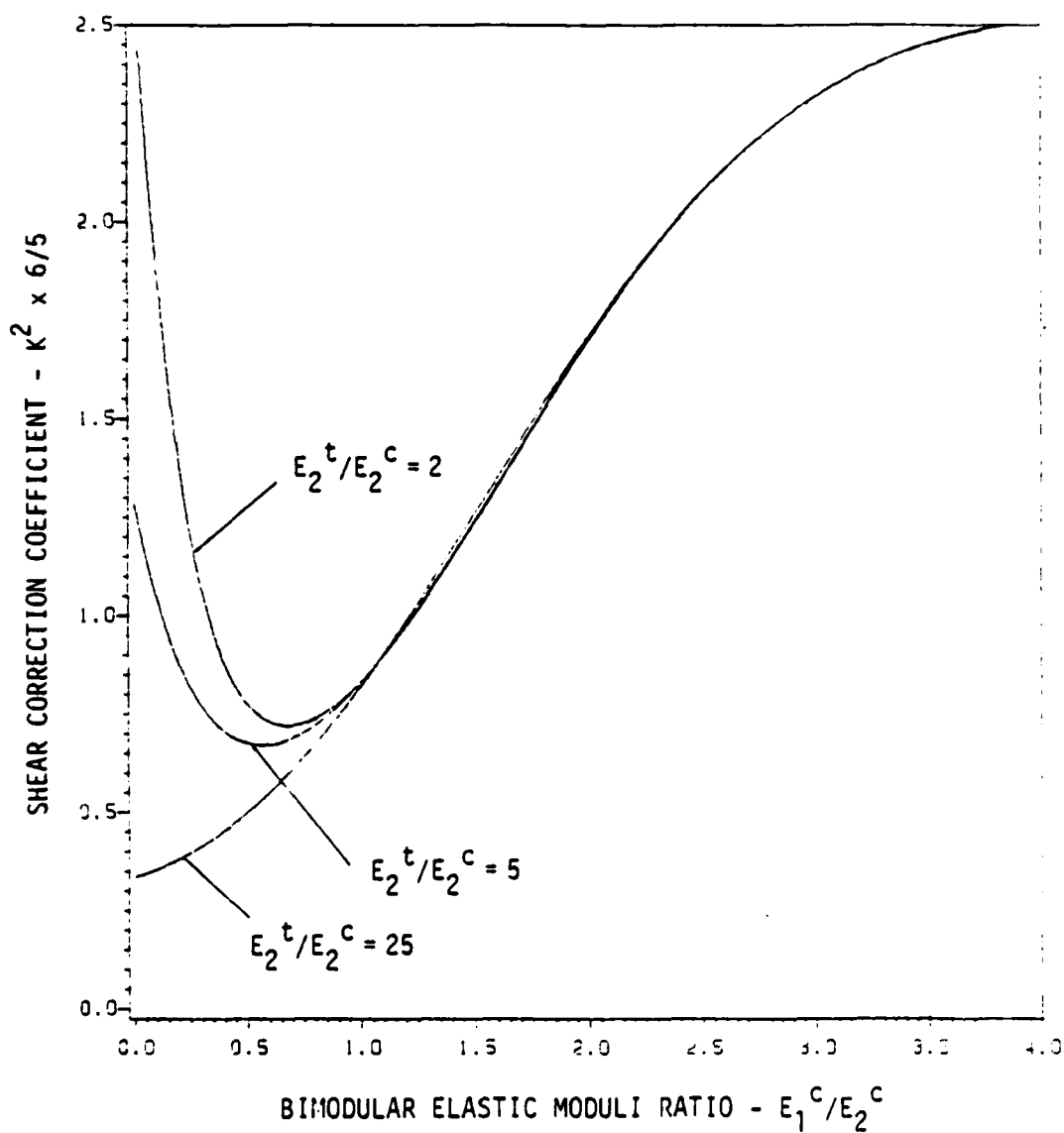


Figure 3. Effect of bimodular elastic moduli ratio E_1^C/E_2^C on shear correction coefficient for a three-layer bimodular laminated ($h_1/h_2=h_3/h_2=1$, $G_1/G_2=G_3/G_2=1$, $E_3^t/E_2^C=2$).

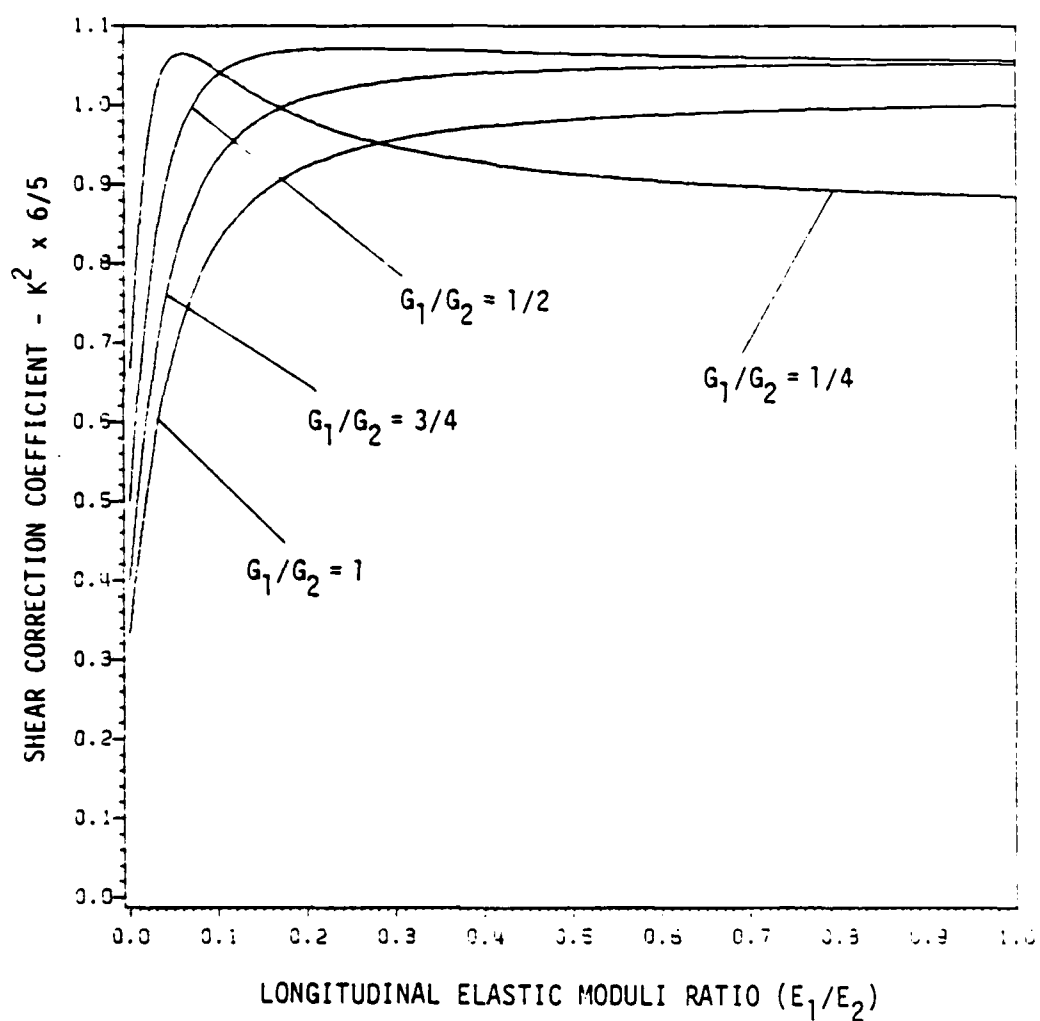


Figure 4. Effect of elastic moduli ratio and shear correction coefficient for various shear-moduli ratios for three-layers unimodular-material laminates ($h_1/h_2 = h_3/h_2 = 1$, $E_1/E_3 = 1$, $G_1/G_3 = 1$).

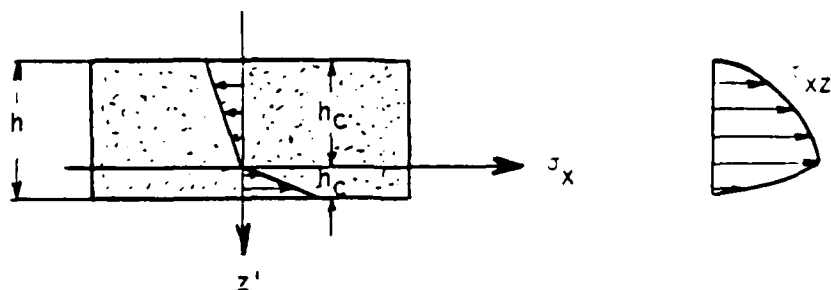


Figure A-1. Normal and shear stress distribution in a single-layer bimodular material.

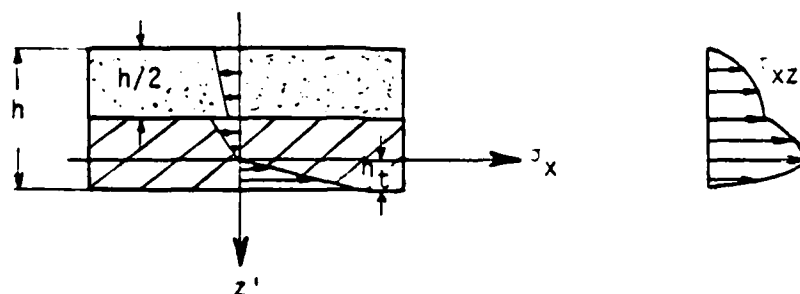


Figure A-2. Normal and shear stress distribution in a two-layer bimodular material laminate.

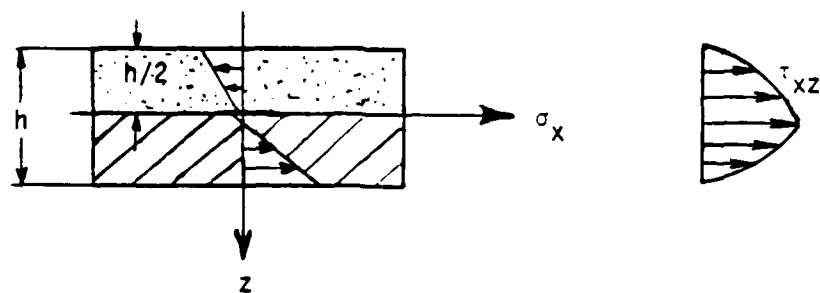


Figure A-3. Normal and shear stress distribution in a two-layer unimodular material laminate.

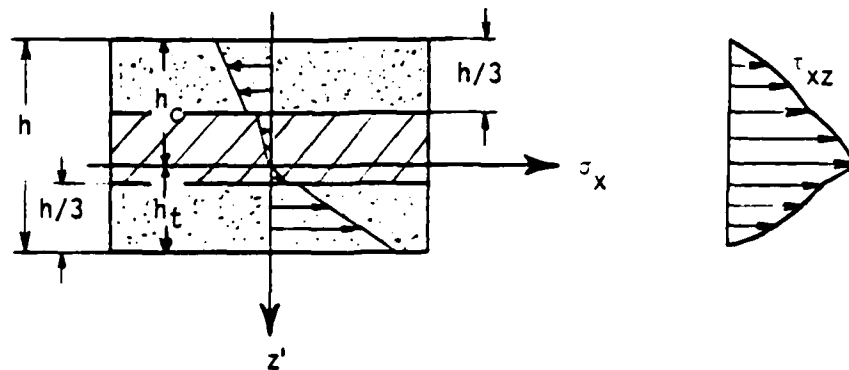


Figure A-4. Normal and shear stress distribution in a three-layer bimodular material laminate.

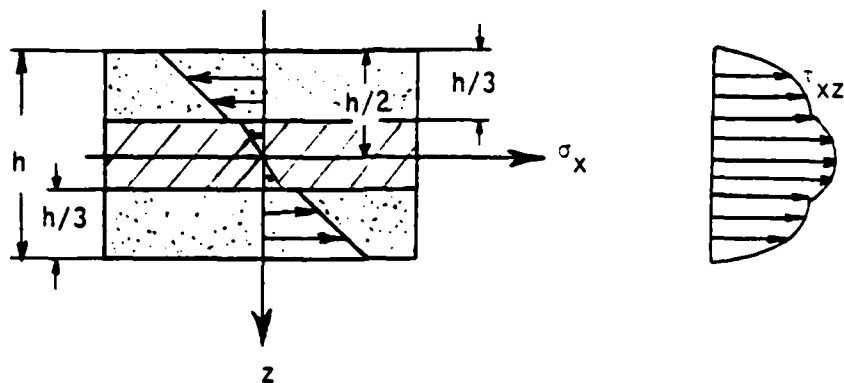


Figure A-5. Normal and shear stress distribution in a three-layer unimodular material laminate.

PREVIOUS REPORTS ON THIS CONTRACT

Project Rept. No.	Issuing University Rept. No.*	Report Title	Author(s)
1	OU 79-7	Mathematical Modeling and Micromechanics of Fiber Reinforced Bimodulus Composite Material	C.W. Bert
2	OU 79-8	Analyses of Plates Constructed of Fiber-Reinforced Bimodulus Materials	J.N. Reddy & C.W. Bert
3	OU 79-9	Finite-Element Analyses of Laminated Composite-Material Plates	J.N. Reddy
4A	OU 79-10A	Analyses of Laminated Bimodulus Composite-Material Plates	C.W. Bert
5	OU 79-11	Recent Research in Composite and Sandwich Plate Dynamics	C.W. Bert
6	OU 79-14	A Penalty Plate-Bending Element for the Analysis of Laminated Anisotropic Composite Plates	J.N. Reddy
7	OU 79-18	Finite-Element Analysis of Laminated Bimodulus Composite-Material Plates	J.N. Reddy & W.C. Chao
8	OU 79-19	A Comparison of Closed-Form and Finite-Element Solutions of Thick Laminated Anisotropic Rectangular Plates	J.N. Reddy
9	OU 79-20	Effects of Shear Deformation and Anisotropy on the Thermal Bending of Layered Composite Plates	J.N. Reddy & Y.S. Hsu
10	OU 80-1	Analyses of Cross-Ply Rectangular Plates of Bimodulus Composite Material	V.S. Reddy & C.W. Bert
11	OU 80-2	Analysis of Thick Rectangular Plates Laminated of Bimodulus Composite Materials	C.W. Bert, J.N. Reddy, V.S. Reddy, & W.C. Chao
12	OU 80-3	Cylindrical Shells of Bimodulus Composite Material	C.W. Bert & V.S. Reddy
13	OU 80-6	Vibration of Composite Structures	C.W. Bert
14	OU 80-7	Large Deflection and Large-Amplitude Free Vibrations of Laminated Composite-Material Plates	J.N. Reddy & W.C. Chao
15	OU 80-8	Vibration of Thick Rectangular Plates of Bimodulus Composite Material	C.W. Bert, J.N. Reddy, W.C. Chao, & V.S. Reddy
16	OU 80-9	Thermal Bending of Thick Rectangular Plates of Bimodulus Material	J.N. Reddy, C.W. Bert, Y.S. Hsu, & V.S. Reddy
17	OU 80-14	Thermoelasticity of Circular Cylindrical Shells Laminated of Bimodulus Composite Materials	Y.S. Hsu, J.N. Reddy, & C.W. Bert
18	OU 80-17	Composite Materials: A Survey of the Damping Capacity of Fiber-Reinforced Composites	C.W. Bert
19	OU 80-20	Vibration of Cylindrical Shells of Bimodulus Composite Materials	C.W. Bert & M. Kumar
20	VPI 81-11 & OU 81-1	On the Behavior of Plates Laminated of Bimodulus Composite Materials	J.N. Reddy & C.W. Bert
21	VPI 81-12	Analysis of Layered Composite Plates Accounting for Large Deflections and Transverse Shear Strains	J.N. Reddy
22	OU 81-7	Static and Dynamic Analyses of Thick Beams of Bimodular Materials	C.W. Bert & A.D. Tran
23	OU 81-8	Experimental Investigation of the Mechanical Behavior of Cord-Rubber Materials	C.W. Bert & M. Kumar
24	VPI 81.28	Transient Response of Laminated, Bimodular-Material Composite Rectangular Plates	J.N. Reddy
25	VPI 82.2	Nonlinear Bending of Bimodular-Material Plates	J.N. Reddy & W.C. Chao
26	OU 82-2	Analytical and Experimental Investigations of Bimodular Composite Beams	C.W. Bert, C.A. Rebello, & C.J. Rebello
27	OU 82-3	Research on Dynamics of Composite and Sandwich Plates, 1979-81	C.W. Bert
28	OU 82-4 & VPI 82.20	Mechanics of Bimodular Composite Structures	C.W. Bert & J.N. Reddy

*OU denotes the University of Oklahoma; VPI denotes Virginia Polytechnic Institute and State University.

Previous Reports on this Contract - Cont'd
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<u>Project Rept. No.</u>	<u>Issuing University Rept. No.</u>	<u>Report Title</u>	<u>Author(s)</u>
29	VPI 82.19	Three-Dimensional Finite Element Analysis of Layered Composite Structures	W.C. Chao, N.S. Putcha, J.N. Reddy
30	OU 82-5	Analyses of Beams Constructed of Nonlinear Materials Having Different Behavior in Tension and Compression	C.W. Bert & F. Gordaninejad
31	VPI 82.31	Analysis of Layered Composite Plates by Three-Dimensional Elasticity Theory	J.N. Reddy & T. Kuppusamy

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18. SUPPLEMENTARY NOTES This work will be presented as an invited paper at 20th Annual Meeting of the Society of Engineering Science, University of Delaware, Newark, Delaware, August 22-24, 1983.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Beams, bending stress, bimodular materials, composites, cylindrical bending, laminates, shear deformation, shear stress, static bending, transverse shear correction coefficient.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A closed-form solution is presented for the transverse shear-stress distri- bution in a laminate constructed of bimodular materials (which have differ- ent elastic moduli in tension and compression). This solution is based upon the axial-force equilibrium equation of two-dimensional elasticity theory. It is used to obtain closed-form expressions for the Timoshenko-type trans- verse shear correction factor (based upon equivalent shear strain energy) and the maximum transverse shear stress. The effect of material (over)		

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20. Abstract - Cont'd

bimodularity (even in the case of a single layer) is to shift the neutral-surface location away from the midplane of the laminate. However, in the case of a single layer, the numerical values of shear correction factor and maximum shear stress are identical to those of an ordinary material. For multiple layers, the shear correction factor and maximum shear stress can be significantly different from those for ordinary-material laminates.

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